Eur. Phys. J. C 23, 357–361 (2002)
Digital Object Identifier (DOI) 10.1007/s100520100872

THE EUROPEAN
PHYSICAL JOURNAL C

Does a Kalb–Ramond field make spacetime optically active?

Sayan Kar[1,a], Parthasarathi Majumdar[2,b], Soumitra SenGupta[3,c], Aninda Sinha[4,d,⋆]

[1] Department of Physics and Centre for Theoretical Studies, Indian Institute of Technology, Kharagpur 721 302, India
[2] The Institute of Mathematical Sciences, CIT Campus, Taramani, Chennai 600 113, India
[3] Department of Physics, Jadavpur University, Calcutta 700 032, India
[4] St. Edmunds College, University of Cambridge, Cambridge CB3 0BN, UK

Received: 27 October 2001 /
Published online: 25 January 2002 –

Abstract. A spacetime with torsion produced by a Kalb–Ramond field coupled gravitationally to the Maxwell field, in accordance with a recent proposal by two of us (PM and SS), is argued to lead to optical activity in synchrotron radiation from cosmologically distant radio sources. We show that this indicates a very small, but possibly observable rotation of the plane of polarization of the radiation, above and beyond the Faraday rotation due to magnetized galactic plasma. Implications for heterotic string theory are outlined.

1 Introduction

The massless antisymmetric tensor Kalb–Ramond (KR) field has been an inherent aspect of supergravity theories. Indeed, the tensor multiplet in $N = 1$ theories has interesting duality properties that are exploited in its coupling to supergravity [1]. In extended supergravity, the KR field becomes a part of the supergravity multiplet itself, thus playing a more intrinsic role. The importance of the KR field in supergravity theories in various spacetime dimensions has been emphasized more than ever in string theories [2]. Supergravity multiplets constitute the massless sector of string theories. As such, they inevitably contain massless KR fields. Such fields have an important property in that they enable one to implement a spacetime background for string theory possessing *torsion* in addition to curvature.

One aspect which particularly deserves attention in this respect is that of cosmology in the presence of torsion, or, equivalently, in the presence of a KR field. Indeed, the cosmological domain is the most likely arena for physical "stringy" effects to appear. Since sources of torsion exist in the massless spectra of most viable string theories (in the form of the KR field), at least in the perturbative sector, cosmological models with non-zero torsion are of substantial interest. Restricting one's attention to Einstein–Cartan spacetimes, the issue of gauge invariant coupling to standard massless gauge fields arises. The well-known problem [3] associated with the Maxwell field has been addressed in [4] by introducing a KR field, and augmenting it in accord with requirements of quantum consistency of heterotic string theory toroidally compactified to four spacetime dimensions.

In this paper, we examine the consequences of the resulting dynamics, to discern effects that could, even if remotely, be astrophysically/cosmologically observable. The possibility that a KR field may induce a rotation of the plane of polarization of electromagnetic synchrotron radiation from cosmologically distant sources, was already alluded to in [4]. Now, it is well known that galactic synchrotron radiation frequently exhibits an optical activity in that the plane of polarization undergoes a rotation during propagation, through an angle proportional to λ^2 (where λ is the wavelength). This is the phenomenon of Faraday rotation that occurs during propagation of electromagnetic waves through a magnetized medium like the galactic magnetized plasma. The constant of proportionality, called the Faraday rotation measure is a function of the galactic (or inter-galactic) magnetic field, the density of the medium and so on.

The key aim of the present paper is to argue that Einstein–Kalb–Ramond–Maxwell coupling leads to an *additional* rotation of the polarization plane, which is *independent* of wavelength, and hence an effect beyond the well-studied Faraday rotation. Furthermore, this rotation is universal in that it is independent of galactic parameters like magnetic field, density of the plasma etc. One expects the angle of rotation to be non-negligible only for galaxies of large (i.e. > 2.0) redshift, so that one is tempted to attribute a truly cosmological origin to this effect.

A remark on our basic strategy is perhaps in order: in the sequel, we treat the KR field as a *tiny* perturbation

[a] e-mail: sayan@cts.iitkgp.ernet.in
[b] e-mail: partha@imsc.ernet.in
[c] e-mail: soumitra@juphys.ernet.in
[d] e-mail: as402@cam.ac.uk
⋆ *Present address:* DAMTP, University of Cambridge, Wilberforce Road, Cambridge, CB3 OWA, UK

on the Maxwell field equations in a standard cosmological background. In other words, despite the assumed "primordial" origin of the KR field, we restrict its strength to such small values that its energy density plays an insignificant role in shaping geometry on a cosmological scale. Put differently, we assume that the KR field has stabilized with respect to the cosmic (matter or radiation) fluid long before photon-dust decoupling. There can exist in principle however a cosmic KR *background* field which has not disappeared entirely, and it is this which purportedly leads to the "universal" optical activity alluded to earlier. Clearly, one expects this to influence the polarization anisotropy of the cosmic microwave background radiation (CMBR) as well, the details of which we hope to report elsewhere.

For the spacetime background, we choose the *spatially* flat Friedmann–Robertson–Walker background with the scale factor depending only on (conformal) time, evolving according to both a radiation dominated and a matter dominated scenario. The optical activity appears to persist in both cases.

This paper is organized as follows: in Sect. 2, we review the important aspects of the earlier paper [4] as background for the present work. This is followed in Sect. 3 by a presentation of the solution of Maxwell–KR field equations in a flat background spacetime and retaining only the leading order coupling of the two fields. The first hint of optical activity already appears at this preliminary, albeit cosmologically unphysical stage. Section 4 constitutes the main part of the work, where, in a spatially flat FRW background, the Maxwell–KR equations are considered for conformal factors pertaining to the radiation and matter dominated scenarios. An expression for the angle of rotation of the plane of polarization is obtained, in the large η (conformal time) dependence on redshift. Section 5 summarizes our conclusions.

2 Einstein–Maxwell–Kalb–Ramond coupling

Let us briefly recapitulate the main tenets of the earlier paper [4]. It is well known that the electromagnetic field tensor, defined as the generally covariant curl of the four potential, is not invariant under the standard $U(1)$ electromagnetic gauge transformation $\delta A_\mu = \partial_\mu \omega$, assuming that the torsion tensor $T^\rho_{\mu\nu}$ – a purely geometric quantity like curvature – must be gauge invariant. Therefore the "minimal coupling prescription" is not tenable in this situation. Thus, the standard definition of the field tensor, viz., $F_{\mu\nu} = \partial_{[\mu} A_{\nu]}$ is retained, and all covariant derivatives in the sequel are defined using the Christoffel connection. However, this does not lead to any coupling of the Maxwell field to the torsion. To that effect, we introduce a Kalb–Ramond (KR) antisymmetric second rank tensor field $B_{\mu\nu}$ as a possible source of torsion. The KR field strength is modified by $U(1)$ Chern–Simons terms which originate from the quantum consistency of an underlying string theory [2]. This augmented field is coupled to the torsion in a way that the resulting action preserves all gauge symmetries and has the form

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[\frac{1}{16\pi G} R(g,T) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \tilde{H}_{\mu\nu\lambda} \tilde{H}^{\mu\nu\lambda} + \frac{1}{\sqrt{G}} T^{\mu\nu\lambda} \tilde{H}_{\mu\nu\lambda} \right], \quad (1)$$

where R is the scalar curvature, defined by $R = R_{\alpha\mu\beta\nu} g^{\alpha\beta} g^{\mu\nu}$. $R_{\alpha\mu\beta\nu}$ is the Riemann–Christoffel tensor:

$$R^\kappa_{\mu\nu\lambda} = \partial_\mu \Gamma^\kappa_{\nu\lambda} - \partial_\nu \Gamma^\kappa_{\mu\lambda} + \Gamma^\kappa_{\mu\sigma} \Gamma^\sigma_{\nu\lambda} - \Gamma^\kappa_{\nu\sigma} \Gamma^\sigma_{\mu\lambda}. \quad (2)$$

The torsion tensor $T_{\mu\nu\lambda}$ is an auxiliary field in (1), obeying the constraint equation

$$T_{\mu\nu\lambda} = \sqrt{G} \tilde{H}_{\mu\nu\lambda}, \quad (3)$$

where $\tilde{H}_{\mu\nu\lambda} \equiv \partial_{[\mu} B_{\nu\lambda]} + (1/3) G^{1/2} A_{[\mu} F_{\nu\lambda]}$ [4]. Thus, the augmented KR field strength three tensor plays the role of the spin angular momentum density which is the source of torsion [5]. Substituting the above equation in (1) and varying with respect to $B_{\mu\nu}$ and A_μ respectively, we obtain the equations

$$D_\mu \tilde{H}^{\mu\nu\lambda} = 0 \quad (4)$$

and

$$D_\mu F^{\mu\nu} = \sqrt{G} \tilde{H}^{\mu\nu\lambda} F_{\lambda\mu}. \quad (5)$$

Now, the KR three tensor is Hodge dual to the derivative of a spinless field ϕ, so that, after a partial integration, one obtains

$$S_{\mathrm{int}} = \frac{1}{2} \int \mathrm{d}^4 x \phi F_{\mu\nu} {}^*F^{\mu\nu}, \quad (6)$$

where ${}^*F^{\mu\nu} \equiv \epsilon^{\mu\nu\lambda\sigma} F_{\lambda\sigma}$. Here, we have noted the fact that

$$\frac{1}{\sqrt{-g}} \partial_\nu (\sqrt{-g}\, {}^*F^{\mu\nu}) = D^S_\mu {}^*F^{\mu\nu} = 0 \quad (7)$$

by the Maxwell–Bianchi identity, where D^S is the covariant derivative using the Christoffel connection.

In general, in addition to the graviton and the KR field, the perturbative sector of the heterotic string contains a scalar dilaton field whose dynamics is also known to have cosmological consequences [6]. In this paper, we shall however ignore this dynamics for the moment and focus instead on what the KR field does. In any event, the dilaton field couples to the Maxwell Lagrangian and the kinetic term of the KR field, and so cannot affect in any major way the optical activity induced by the axion (KR) field; the latter effects appear due to the fact that F^*F is *pseudoscalar*. Freezing the dilaton field and taking the KR field strength to be

$$H_{\mu\nu\rho} = \epsilon^\sigma_{\mu\nu\rho} D_\sigma H,$$

where H is a pseudoscalar, one obtains the modified generally covariant Maxwell equations [4]

$$\begin{aligned} \boldsymbol{D} \cdot \boldsymbol{E} &= 2\sqrt{G} \boldsymbol{D} H \cdot \boldsymbol{B}, \\ D_0 \boldsymbol{E} - \boldsymbol{D} \times \boldsymbol{B} &= -2\sqrt{G} [D_0 H \boldsymbol{B} - \boldsymbol{D} H \times \boldsymbol{E}] \\ &\quad + 2G \Big[(\boldsymbol{B}^2 - \boldsymbol{E}^2) \boldsymbol{A} + (\boldsymbol{A} \cdot \boldsymbol{E}) \boldsymbol{E} \\ &\quad + (\boldsymbol{A} \cdot \boldsymbol{B}) \boldsymbol{B} \Big], \\ D_0 \boldsymbol{B} + \boldsymbol{D} \times \boldsymbol{E} &= 0 = \boldsymbol{D} \cdot \boldsymbol{B}. \end{aligned} \quad (8)$$

Here D_μ is the covariant derivative in the spatially flat FRW metric. To a first approximation, we drop the $O(G)$ terms arising from the stringy augmentation of the KR field strength in terms of the Chern–Simons three form. Next, redefine the pseudoscalar field H to absorb the $G^{1/2}$, so that this field becomes dimensionless. The equations now look like

$$\begin{aligned} \boldsymbol{D}\cdot\boldsymbol{E} &= 2\boldsymbol{D}H\cdot\boldsymbol{B},\\ D_0\boldsymbol{E}-\boldsymbol{D}\times\boldsymbol{B} &= -2[D_0H\boldsymbol{B}-\boldsymbol{D}H\times\boldsymbol{E}],\\ D_0\boldsymbol{B}+\boldsymbol{D}\times\boldsymbol{E} &= 0=\boldsymbol{D}\cdot\boldsymbol{B}. \end{aligned} \tag{9}$$

The last equations in the array constitute the Maxwell–Bianchi identity. In a spatially flat isotropic FRW background with metric

$$\mathrm{d}s^2 = R^2(\eta)(\mathrm{d}\eta^2-\mathrm{d}\boldsymbol{x}^2), \tag{10}$$

where η is the conformal time coordinate, defined by $\mathrm{d}\eta = \mathrm{d}t/R(t)$, the above equations assume the form

$$\begin{aligned} \boldsymbol{\nabla}\cdot\boldsymbol{E}R^2 &= 2\boldsymbol{\nabla}H\cdot\boldsymbol{B}R^2,\\ \partial_\eta(\boldsymbol{E}R^2)-\boldsymbol{\nabla}\times\boldsymbol{B}R^2 &= -2[\partial_\eta H\boldsymbol{B}R^2-\boldsymbol{\nabla}H\times\boldsymbol{E}R^2],\\ \partial_\eta(\boldsymbol{B}R^2)+\boldsymbol{\nabla}\times\boldsymbol{E}R^2 &= 0=\boldsymbol{\nabla}\cdot\boldsymbol{B}R^2. \end{aligned} \tag{11}$$

3 Flat universe

We first consider the simple situation corresponding to a *flat* background spacetime ($R(\eta)=1$), just to obtain a preliminary understanding of the effects involved. This simplification does not in any way reduce the qualitative aspects of the optical effects under discussion, although the quantitative details obtained in this manner may not be reliable. Recall that the KR field strength $H_{\mu\nu\rho}=\partial_{[\mu}B_{\nu\rho]}$, so that it satisfies the Bianchi identity

$$\epsilon^{\mu\nu\lambda\sigma}\partial_\sigma H_{\mu\nu\lambda}=0. \tag{12}$$

This immediately implies that the pseudoscalar H satisfies the massless Klein–Gordon equation $\Box H=0$. For non-flat backgrounds, the d'Alembertian operator is to be replaced by its generally covariant counterpart. We note that this is a departure from approaches in the literature where the axion (KR) field H is introduced ad hoc with no specified dynamics; here the Bianchi identity of the dual KR field is precisely the equation of motion of the axion. Assume now that H is only a function of the comoving time coordinate η, so that, the Klein–Gordon equation reduces to the simple equation $\mathrm{d}^2H/\mathrm{d}\eta^2=0$ with the obvious solution $H=h\eta+h_0$, where h and h_0 are constants. Proceeding along the lines of [7,8], we arrive at the equation

$$\frac{\mathrm{d}^2b_\pm}{\mathrm{d}\eta^2}+(k^2\mp 2hk)b_\pm=0, \tag{13}$$

where we have decomposed $\boldsymbol{B}=\boldsymbol{b}(\eta)\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}$ and have chosen the z direction to be the propagation direction of the electromagnetic wave. The circular polarization states are defined by $b_\pm \equiv b_x \pm \mathrm{i}b_y$. Unlike the corresponding equation in [7], (13) can be solved *exactly*:

$$b_\pm = b_0\mathrm{e}^{\mathrm{i}\omega_\pm\eta}\equiv b_0\mathrm{e}^{\mathrm{i}\phi_\pm}, \tag{14}$$

where $\omega_\pm^2\equiv k(k\mp 2h)$.

The optical activity due to the presence of the KR field is then given by the difference

$$(\Delta\phi)_{\mathrm{mag}}\equiv\frac{1}{2}(\phi_+-\phi_-)=-h\eta,\quad \text{for } k\gg h. \tag{15}$$

We also note that the equation for the electric field (with the assumption $\boldsymbol{E}=\boldsymbol{e}(\eta)\mathrm{e}^{\mathrm{i}kz}$ and a similar definition for $e_\pm$) yields an identical result for the amount of rotation of the electric field.

All this is clear indication of an optical activity induced by the KR field. Admittedly, an axion field linearly increasing with time is extremely unphysical from a cosmological point of view, because for large η its energy is bound to affect the cosmological evolution. However, this flat spacetime analysis is to be construed merely as a qualitative illustration of the claim that there is indeed a non-vanishing rotation of the plane of polarization of electromagnetic radiation. Quantitative details of this section are not very relevant from the standpoint of realistic cosmology. However, it will turn out that the qualitative effect does persist even in more realistic cosmological backgrounds.

4 Spatially flat FRW universe: radiation and matter dominated cases

The next immediate step in our analysis is to solve Maxwell equations once again in a non-trivial cosmology – we choose for simplicity the *spatially* flat Friedmann–Robertson–Walker (FRW)-type of background. The equation for the pseudoscalar field is given by

$$\Box H=0. \tag{16}$$

For a spatially independent H field, such that $H\equiv H(\eta)$ we have a first integral of the form

$$\partial_0 H=\frac{h}{R^2(\eta)}, \tag{17}$$

where h is an integration constant, which, in a sense, is a "measure" of the pseudoscalar H field or, equivalently, the dual three form field $H_{\mu\nu\lambda}$.

The equations that the polarization states $b_\pm$ satisfy for such a background can be similarly written down in terms of the quantity $F_\pm$, where $b_\pm=F_\pm/R^2$. They are

$$\frac{\mathrm{d}^2F_\pm}{\mathrm{d}\eta^2}+\left(k^2\mp\frac{2hk}{R^2(\eta)}\right)F_\pm=0. \tag{18}$$

A corresponding equation for the electric field polarisation states $e_\pm$ can also be obtained in terms of the quantity

$G_\pm$, where $e_\pm = G_\pm/R^2$, which is identical to the equation for $F_\pm$.

The equations, of course, can be solved explicitly only if knowledge of the scale factor $R(\eta)$ is available. One may resort to a WKB approximation along the lines of [7] and arrive at qualitative results. We prefer, as an alternative exercise, to choose a "physically reasonable" scale factor and derive the effects of optical activity for such a case. As mentioned earlier, the actual situation will correspond to a scale factor corresponding to a solution of the Einstein–KR–Maxwell-dilaton equations of motion for low energy effective supergravity. However, since the dilaton couples to the Maxwell Lagrange density and not to F^*F, it is unlikely that the effect that we find without it will be washed away by its inclusion.

Let us assume a scale factor $R(\eta) = \eta/\eta_0^R$ which is equivalent, in real time, to the scale factor of a radiation dominated FRW model, for $1/\eta_0^R = (8\pi G\epsilon_0/3)^{1/2}$, with ϵ_0 being the primordial radiant energy density. For a matter dominated model we assume $R(\eta) = \left(\eta/\eta_0^M\right)^2$. Our objective is to obtain the *asymptotic* dependence on η and the parameters of the theory which are $\tilde{h} = h(\eta_0^R)^2$, $h' = h(\eta_0^M)^4$ and the wave number k. Accordingly, we have the two equations for the radiation and matter dominated cases which we quote below:

$$\frac{\mathrm{d}^2F_\pm}{\mathrm{d}x^2} + \left(1 - \frac{\mu_\pm^2}{x^2}\right)F_\pm = 0, \quad \mu_\pm^2 \equiv 2\tilde{h}k, \tag{19}$$

and

$$\frac{\mathrm{d}^2F_\pm}{\mathrm{d}x^2} + \left(1 - \frac{\mu_\pm^2}{x^4}\right)F_\pm = 0, \quad \mu_\pm^2 \equiv 2h'k^3. \tag{20}$$

In the above, we use dimensionless quantities throughout, with $x = k\eta$.

We now use the ansatz

$$F_\pm(x) = \mathrm{e}^{\mathrm{i}x} v_\pm(x), \tag{21}$$

so that (19) and (20) reduce to

$$\frac{\mathrm{d}^2v_\pm}{\mathrm{d}x^2} + 2\mathrm{i}\frac{\mathrm{d}v_\pm}{\mathrm{d}x} - \frac{\mu_\pm^2}{x^2}v_\pm = 0, \tag{22}$$

$$\frac{\mathrm{d}^2v_\pm}{\mathrm{d}x^2} + 2\mathrm{i}\frac{\mathrm{d}v_\pm}{\mathrm{d}x} - \frac{\mu_\pm^2}{x^4}v_\pm = 0. \tag{23}$$

We are only interested in an asymptotic solution of these equations for $x \to +\infty$. Accordingly, we choose a solution for both cases of the type

$$v_\pm(x) = v_0^\pm + \frac{v_1^\pm}{x} + \frac{v_2^\pm}{x^2} + \cdots \tag{24}$$

This is an asymptotic solution, as given in standard texts on differential equations. If this ansatz is used to calculate the various coefficients, one finds that in both cases *all* coefficients are proportional to $v_0^\pm$. For the radiation dominated case, the coefficients are a finite series in powers of $\tilde{h}$; for the matter dominated case, they are all proportional to h'. The angle of rotation of the polarization plane, to lowest non-trivial order in $1/x$ (remember that $x = k\eta$) is given by

$$\Delta\phi = |\arg v_0^+ - \arg v_0^- + 2\tan^{-1}(\tilde{h}k/x)|, \quad \text{for RD}, \tag{25}$$

and

$$\Delta\phi = |\arg v_0^+ - \arg v_0^- + 2\tan^{-1}(h'k^3/3x^3)|, \quad \text{for MD}. \tag{26}$$

But, recalling that the angle of rotation must vanish in the absence of our proposed interaction, we get $\arg v_0^+ - \arg v_0^- = 0$. Note that *no* assumption has been made about the dependence of the coefficients on h. Thus, the angle of rotation in the two cases are

$$\Delta\phi = |2\tan^{-1}(\tilde{h}/\eta)|, \quad \text{for } RD, \tag{27}$$

and

$$\Delta\phi = |2\tan^{-1}(h'/3\eta^3)|, \quad \text{for } MD. \tag{28}$$

For very small h, the inverse tangent may be replaced by its argument. These are our predictions (to the lowest order in h) for the rotation angle. It is more convenient to replace the co-moving time coordinate η first in terms of t defined earlier (just following (10)), and then reexpressing t in terms of the redshift z,

$$t - t_0 = \frac{1}{2H_0}\left[1 - (1+z)^{-2}\right], \quad \text{for } RD, \tag{29}$$

$$t - t_0 = \frac{2}{3H_0}\left[1 - (1+z)^{-3/2}\right], \quad \text{for } MD. \tag{30}$$

One can obtain expressions for the rotation of the electric field which turn out to be the same as for the magnetic field. The resulting formulae can be considered to be the predictions of our model for the rotation in the plane of polarization of synchrotron radiation from cosmologically distant galaxies, above and beyond the standard Faraday rotation due to galactic (and possibly inter-galactic) magnetic fields.

5 Conclusions

In this paper, we have presented arguments to the effect that due to a KR (axion) field which endows the spacetime in its immediate vicinity with torsion, one arrives at a "primordial" optical activity. The rotation of the polarization plane of electromagnetic radiation from cosmologically distant galaxies is predicted to be
(a) independent of wavelength and
(b) "universal" in the sense of its independence of parameters of the source galaxy (or indeed the inter-galactic medium). An observational result in support of this prediction is most likely evidence of the existence of a primordial KR field and hence of a spacetime with (non-propagating) torsion in an early epoch. Since a KR field occurs naturally in some supergravity theories and hence in the massless spectrum of closed string theory, such an observation may perhaps be construed to be evidence for

supergravity as well as being a hint of an underlying string structure. There is also the likelihood of a KR field being a component of dark matter[1]. As pointed out in [12], such a weak coupling could be detected perhaps in future, if not through presently available data.

We should also remark that our results imply the absence of any inherent anisotropy of spacetime. The origin of the effect, as stated earlier, is the additional KR field coupling to the Maxwell field with gravitational strengths in a manner manifestly consistent with an isotropic background spacetime. This is in contrast to an hypothesis [9] behind the claim of an observed effect, which postulated a preferred direction in space around which the data ostensibly displayed a dipole-type angular distribution. We do not predict any such correlations.

As already alluded to in the Introduction, the optical activity we discuss here is also likely to be manifest in the CMBR, that is to say in its polarization anisotropy. This is already on the agenda of observational astronomy, most notably in the latest microwave anisotropy probe (MAP) launched recently by NASA. The measurements of the anisotropy in the polarization map of CMBR is an important aspect of the program undertaken in MAP. We hope to report in the near future on the ramifications of our work for such an anisotropy.

Apart from the astrophysical ramifications of our work, the fact that the only known proposal of coupling the Maxwell field to an Einstein–Cartan geometry in a gauge invariant manner leads directly to the optical activity discussed above can, in principle, be of significant use in the detection of torsion as a geometrical property of spacetime.

Acknowledgements. We acknowledge useful correspondence with P. Jain. SK thanks S. Bharadwaj for useful discussions. AS acknowledges the local hospitality provided by the Institute of Mathematical Sciences, Chennai, India where part of this work was done. PM would like to thank J. Ambjorn, A. Ashtekar, S. Chaudhuri and H. Nicolai for discussions, and the Centre for Gravitational Physics, Penn State University, USA, The Niels Bohr Institute, Copenhagen, Denmark and the Albert Einstein Institute, Potsdam, Germany, for hospitality during the completion of this work. SK and PM thank J. Bagchi and T. Saini for informative discussions pertaining to observations.

References

1. S.J. Gates, M. Grisaru, M. Rocek, W. Siegel, Superspace (W.A. Benjamin, New York 1983)
2. M. Green, J. Schwarz, E. Witten, Superstring theory, vol. II (Cambridge 1985)
3. R. Hammond, Gen. Rel. Grav. **23**, 1195 (1991)
4. P. Majumdar, S. SenGupta, Class. Quant. Grav. **16**, L89 (1999)
5. F. Hehl, P. von der Heyde, G. Kerlick, J. Nester, Rev. Mod. Phys. **48**, 393 (1976)
6. G. Veneziano, Les Houches Lectures, July, 1999, hep-th/0002094; M. Gasperini, hep-th/0004149
7. S. Carroll, G. Field, Phys. Rev. D **43**, 3789 (1991)
8. S. Carroll, G. Field, R. Jackiw, Phys. Rev. D **41**, 1231 (1990)
9. B. Nodland, J.P. Ralston, Phys. Rev. Lett. **78**, 3043 (1997); Phys. Rev. Lett. **79**, 1958 (1997); astro-ph/9708114; astro-ph/9706126
10. P. Jain, J. Ralston, Mod. Phys. Lett. A **14**, 417 (1999)
11. D. Eisenstein, E. Bunn, Phys. Rev. Lett. **79**, 1957 (1997); S. Carroll, G. Field, Phys. Rev Lett **79**, 2394 (1997); J. Leahy, astro-ph/9704285; J. Wardle, R. Perley, M. Cohen, Phys. Rev. Lett. **79**, 1801 (1997); T. Loredo, E. Flannagan, I. Wassermann, Phys. Rev. D **56**, 7507 (1997)
12. S. Carroll, Phys. Rev. Lett. **81**, 3067 (1998)

[1] There are claims in the literature on observational astrophysics of evidence that optical activity of a related type may have already been *observed* in radiation from distant quasars and other radio sources [9,10]. However, such claims have been strongly refuted by others as being statistically insignificant [11]. We do not adjudicate on this controversy here